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The Simple & The Complex

• Complexity?
The Simple & The Complex

Mondrian
The Simple & The Complex

$H = 0$

$C = 0$

*Mondrian*
The Simple & The Complex

Pollock
The Simple & The Complex

$H = 1$

$C = 0$

Pollock
The Simple & The Complex

Bosch
The Simple & The Complex

$H \neq 0$

$C \neq 0$

_Bosch_
<table>
<thead>
<tr>
<th>H = 0</th>
<th>H ≠ 0</th>
<th>H = 1</th>
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<tr>
<td>C = 0</td>
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The COMPLEXITY has to do with intricate structures hidden in the dynamics, emerging from a system which itself is much simpler than its dynamics. Complexity is characterized by the paradoxical situation of complicated dynamics of simple systems.

- Periodic motion it is not complex.
- White noise it is not complex.
# Crystal & Ideal Gas

<table>
<thead>
<tr>
<th>Crystal</th>
<th>Ideal Gas</th>
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<tbody>
<tr>
<td>High ordered system</td>
<td>Completely disordered system</td>
</tr>
<tr>
<td>Minimal information stored in the system</td>
<td>Maximal information stored in the system.</td>
</tr>
<tr>
<td>Probability distribution in phase space:</td>
<td>Probability distribution in phase space:</td>
</tr>
<tr>
<td>[ P_j = 1 \quad \text{for} \quad j = k ]</td>
<td>[ P_j = \frac{1}{N} \quad \text{for} \quad j = 1, \ldots, N ]</td>
</tr>
<tr>
<td>[ P_j = 0 \quad \text{for} \quad j \neq k ]</td>
<td></td>
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<tr>
<td>Maximum Disequilibrium</td>
<td>Minimum Disequilibrium</td>
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</table>
Statistical Complexity

\[ C = H \cdot Q \]

Diagram:
- Information = H
- Complexity = C = H + D
- Disequilibrium = D
- Crystal to Ideal Gas transition
Disorder $H$

- We define for a given probability distribution

$$P = \{p_j, \ j = 1, \cdots, N\} \in \Omega \subset \mathbb{R}^N$$

and its associate information measure $\mathcal{I}[P]$, an amount of “disorder” $H$ in the fashion

$$H[P] = \frac{\mathcal{I}[P]}{\mathcal{I}_{\text{max}}} ,$$

where $\mathcal{I}_{\text{max}} = \mathcal{I}[P_e]$ and $P_e$ is the probability distribution which maximize the information measure, where $P_e$ is the equilibrium probability distribution. Then $0 \leq H \leq 1$. 
We define the “disequilibrium” adopting some kind of distance from the equilibrium distribution $P_e$ of the accessible states of the system.

$$Q[P] = Q_0 \{D[P, P_e]\},$$

where $Q_0$ is a normalization constant and $0 \leq Q \leq 1$. The disequilibrium $Q$ would reflect on the system’s “architecture”, being different from zero if there are “privileged”, or more likely states among the accessible ones.
Selection of the information measure $I$

- **Shannon Entropy:**

$$S_S[ P ] = - \sum_i p_i \cdot \ln p_i .$$

- **Tsallis Entropy:**

$$S_T^{(q)}[ P ] = \frac{1}{q-1} \left[ 1 - \sum_i (p_i)^q \right] .$$

- **Escort-Tsallis Entropy:**

$$S_G^{(q)}[ P ] = \frac{1}{q-1} \left[ 1 - \left\{ \sum_i (p_i)^{1/q} \right\}^{-q} \right] .$$

- **Rényi entropy:**

$$S_R^{(q)}[ P ] = \frac{1}{(1-q)} \ln \left\{ \sum_{j=1}^N (p_j)^q \right\} .$$
Selection of Distance D

- **Euclidean distance:**
  \[ D_E[P, P_e] = \| P - P_e \|_E = \sum_{j=1}^{N} \left( p_j - \frac{1}{N} \right)^2. \]

- **Wootters distance:**
  \[ D_W[P_1, P_2] = \cos^{-1} \left\{ \sum_{j=1}^{N} \left( p_j^{(1)} \right)^{1/2} \cdot \left( p_j^{(2)} \right)^{1/2} \right\}. \]

- **Relative entropy (Kullback relative entropy):**
  \[ D_{K_q}^\kappa[P, P_e] = K_q^{(\kappa)}[P|P_e] = S_q^{(\kappa)}[P_e] - S_q^{(\kappa)}[P]. \]

- **Jensen divergence:**
  \[ D_{J_q}^\kappa[P, P_e] = \mathcal{J}_{S_q}^{1/2}[P, P_e] = \]
  \[ = \frac{1}{2} K_q^{(\kappa)} \left[ P \mid \frac{P + P_e}{2} \right] + \frac{1}{2} K_q^{(\kappa)} \left[ P_e \mid \frac{P + P_e}{2} \right]. \]
The family of Statistical Complexity Measures, $C^{(\kappa)}_{\nu,q}$, is defined by

$$C^{(\kappa)}_{\nu,q}[P] = H^{(\kappa)}_q[P] \cdot Q^{(\nu)}_q[P]$$

This quantity reflects on the interplay between the amount of information stored in the system and its disequilibrium.

- $\kappa = S, T, G, R$: Shannon, Tsallis, Generalized Escort-Tsallis, Rényi, for a fixed $q$.
  
  In Shannons instance ($\kappa = S$) we have, of course, $q = 1$.
- $\nu = E, W, K, J$: Euclidean, Wootters, Kullback, Jensen.
(i) Probability subspace $\Omega$ for $N = 4$: $\Omega \equiv \Delta^3$ (3-simplex) in an hyperplane of dimension 3. Dotted lines effect the barycentric subdivision with $\mu_3$ the $\Omega$-barycenter.

(ii) Sub-simplex $\Delta_3^3$.

(iii) Maximum and minimum of complexity as function of $H$ obtained by consecutive borders of the sub-simplex $\Delta_3^3$. 
The Logistic Map, \( F : x_n \rightarrow x_{n+1} \) is described by the ecologically motivated, dissipative system described by the first order difference equation

\[
x_{n+1} = r \cdot x_n \cdot (1 - x_n)
\]

with \( 0 \leq x_n \leq 1 \) and \( 0 < r \leq 4 \).

Binary treatment (symbolic dynamics) of the logistic map:
For each parameter value, \( r \), the dynamics of the logistic map was reduced to a binary sequence (0 if \( x \leq \frac{1}{2} \); 1 if \( x > \frac{1}{2} \)) and binary strings of length 12 were considered as states of the system. The concomitant probabilities are assigned according to the frequency of occurrence after running over at least \( 2^{22} \) iterations.
Application to Logistic Map
Application to Logistic Map

Notice that, for the case of periodic windows, if $H < \mathcal{H} \approx 0.3$, we can ascertain that $\lambda < 0$, while for $H > \mathcal{H}$ we see that $\lambda > 0$, which entails chaotic behavior. The LMC statistical complexity is larger for periodic than for chaotic motion, which is wrong!. The Jensen-Shannon statistical complexity measure, $C_{JS}$, on the other hand, behaves in opposite manner, and is also different for distinct degrees of periodicity.
Summing up: the Jensen-Shannon statistical complexity measure \( i \) becomes intensive, \( ii \) is able to distinguish among distinct degrees of periodicity, and \( iii \) yields a better description of dynamical features (a better grasp of dynamical details).
Application to Logistic Map
Application to Logistic Map

N = 6
Given a time series

\[ X = \{x_j, \; j = 1, \cdots, N\} \in \mathbb{R}^N \]

we can define the associate probably distribution function based on

- Histogram of amplitudes.
- Binary representation.
- Frequency (Fourier Transform).
- Frequency bands (Wavelet Transform).
- Ordinal Patterns (Attractor representation).
**Probability distribution**

**Band-Pompe Methodology:**

Given the time-series \( \{x_t, t = 1, \ldots, T\} \) and an embedding dimension \( d > 1 \), we are interested in **ordinal patterns** of order \( d \) generated by

\[
(s) \mapsto (x_{s-(d-1)}, x_{s-(d-2)}, \ldots, x_{s-1}, x_s)
\]

which assign to each time \( s \) the \( d \)-dimensional vector of values at times \( s, s-1, \ldots, s-(d-1) \).

Clearly, the greater the \( d \)-value, the more information on the past our vectors are able to yield.

By the **ordinal pattern** related to the time \( s \) we mean the permutation \( \pi = (r_0, r_1, \ldots, r_{d-1}) \) of \( (0, 1, \ldots, d-1) \) defined by

\[
x_{s-r_{d-1}} \leq x_{s-r_{d-2}} \leq \cdots \leq x_{s-r_1} \leq x_{s-r_0}
\]

**Note that the underlying probability distribution is “extracted” by appropriate consideration regarding causal effects in the system’s dynamics.**
Case $D_3 = 3$, the number of patterns will be $D_3! = 3! = 6$. Graphically one have:

123  132  213  312  231  321
Although being of a quite different physical origin, time series arising from \textit{chaotic systems} share with those generated by \textit{stochastic processes} several properties that make them almost undistinguishable:

- a wide-band power spectrum,
- power spectrum of type $1/f^k$,
- a delta-like autocorrelation function,
- an irregular behavior of the measured signals, etc.
Chaos, Noise & Noise 1 / fk

**Logistic Map:**

\[ X(n+1) = R \, X(n) \, (1 - X(n)) \; ; \; R = 4 \]

\[ H_S = 0.638781 \quad C^{(MPR)}_{JS} = 0.477845 \]
Chaos, Noise & Noise $1/f^k$

Noise $1/f^k : k = 2$

$H_S = 0.858 \quad C_{JS}^{(MPR)} = 0.232$
Chaotic systems:

- The Logistic Map defined by:

  \[ x_{n+1} = r \, x_n \, (1 - x_n) \, . \]

  Note that for \( r = 4 \) this map has a non uniform natural invariant probability density function (PDF).

- The Skew Tent Map: one has

  \[ \begin{cases} 
  x/\omega & \text{for } x \in [0, \omega] \\
  (1-x)/(1-\omega) & \text{for } x \in [\omega, 1] 
  \end{cases} \, . \]

  For any \( \omega \)-value this map has a uniform natural invariant PDF (\( \omega = 0.1847 \) is here considered).

- Henon’s Map: it is a 2D extension of the Logistic Map given by:

  \[ \begin{cases} 
  x_{n+1} = 1 - a \, x_n^2 + y_n \\
  y_{n+1} = b \, x_n 
  \end{cases} \, . \]

  The values used here, \( a = 1.4 \) and \( b = 0.3 \), correspond to a chaotic attractor with a non-smooth PDF.
• The Lorenz Map of Rossler’s oscillator: for the 3D continuous Rossler oscillator one has
\[
\begin{align*}
\dot{x} &= -y - z \\
\dot{y} &= x + ay \\
\dot{z} &= b + z(x - c)
\end{align*}
\]
where \(a = 0.2\), \(b = 0.2\), and \(c = 5.7\) correspond to a chaotic attractor. The Lorenz map is obtained by storing only \(x\)–minimal values.

• Schuster Maps: Schuster and coworkers introduced a class of maps which generate intermittent signals with chaotic bursts that also display \(1/f^z\) noise
\[
\tilde{x}_{n+1} = \tilde{x}_n + \tilde{x}_n^z, \quad \text{Mod 1.}
\]
In particular, results for \(z = 5/2, 2\) and \(3/2\) are reported.
**Stochastic processes:**

- **Noises with \( f^{-k} \) Power Spectrum.** The ensuing time series \( x_i \) has the desired PS and, by construction, is representative of non-Gaussian noises.

- **Fractional Brownian motion (fBm) and fractional Gaussian noise (fGn):** fBm is the only family of processes which is (a) Gaussian, (b) self-similar, and (c) endowed with stationary increments. The fBm and fGn are continuous but non-differentiable processes (in the classical sense). As a non-stationary process, they do not possess a spectrum defined in the usual sense; however, it is possible to define a *generalized power spectrum* of the form: \( \Phi \propto |f|^{-\alpha} \), with \( \alpha = 2\mathcal{H} + 1 \), \( 1 < \alpha < 3 \) for fBm and, \( \alpha = 2\mathcal{H} - 1 \), \( -1 < \alpha < 1 \), for fGn.

Hurst’s \( \mathcal{H} \) parameter defines two distinct regions in the interval \((0,1)\). When \( \mathcal{H} > 1/2 \), consecutive increments tend to have the same sign so that these processes are *persistent*. For \( \mathcal{H} < 1/2 \), on the other hand, consecutive increments are more likely to have opposite signs, and we say that they are *anti-persistent*. 
Chaos, Noise & Noise 1 / f^k

![Graph showing MPR-Complexity vs Entropy for different systems with annotations for skew tent map, Schuster map, Henon map, K-noise, fBm, Rossler minima, and fGn.](image)
Other Applications

- Characterization of laser propagation through turbulent media.
- Encryption test of pseudo-aleatory messages embedded on chaotic laser signals.
- fBm and fGn dynamics.
- Characterization of Pseudo Random Number Generators and randomization of chaotic series.
- Stochastic resonance and coherent resonance.
- Analysis of Classical-Quantum transition problem.
- Study of mode and genre of literary texts. Internal chronological development of authorial styles.
- Alterations in the erythrocyte due to different illness.
- Brain electrical activity: EEG background distinction, epileptic activity, sleep, ERP, EP.
- Quantitative brain maturation.
- Identification of biomarkers that correlate with cancer progression.
Thanks !!!!!

... and see you in Newcastle, Australia
ENTROPY AND STATISTICAL COMPLEXITY PUBLICATIONS:

Permutation entropy of fractional Brownian motion and fractional Gaussian noise.

Randomizing nonlinear maps via symbolic dynamics.

O. A. Rosso, R. Vicente, C. Mirasso

Extracting features of Gaussian self-similar stochastic processes via the Bandt and Pompe approach.

Distinguishing noise from chaos.
O. A. Rosso, H. A. Larrondo, M. T. Martin, A. Plastino, M. A. Fuentes

Bandt-Pompe approach to the classical-quantum transition.
A. M. Kowalski, M. T. Martin, A. Plastino, O. A. Rosso

Characterization of gaussian self-similar stochastic processes using wavelet-based informational tools.
L. Zunino, D. G. Perez, M. T. Martin, A. Plastino, M. Garavaglia, O. A. Rosso
Physical Review E 75 (2007) 021115

Wavelet Jensen-Shannon divergence as a tool for studying the dynamics of frequency band components in EEG epileptic seizures.
M. Pereyra, P. W. Lamberti and O. A. Rosso

Wavelet entropy of stochastic processes.
L. Zunino, D. G. Perez, M. Garavaglia, O. A. Rosso

Model-free stochastic processes studied with q-wavelet-based informational tools.
D. G. Perez, L. Zunino, M. T. Martin, M. Garavaglia, A. Plastino, O. A. Rosso

Entropy changes in brain function.
O. A. Rosso
Alterations of thalassemic erythrocytes detected by wavelet entropy.
A. M. Korol, M. J. Rasia, O, A, Rosso

Random number generators and causality.
H. A. Larrondo, M. T. Martin, C. M Gonzalez, A. Plastino, O. A. Rosso

EEG analysis using wavelet-based informational tools.
O. A. Rosso, M. T. Martin, A. Figliola, K. Keller, A. Plastino

Generalized statistical complexity measures: geometrical and analytical properties.
M.T. Martin, A. Plastino, O. A. Rosso

Wavelet entropy and fractional Brownian motion time series.
D. G. Perez, L. Zunino, M. Garavaglia, O. A. Rosso

Characterization of laser propagation through turbulent media by quantifiers based on wavelet transform: dynamical study.
L. Zunino, D. G. Perez, M. Garavaglia, O. A. Rosso

Entropy and statistical complexity in brain activity.
A. Plastino, O. A. Rosso
Special Number on: "Nonextensive Statistical Mechanics - Theory and Applications",
J. P. Boon, C. Tsallis (Guest Editors).

Entropic non-triviality, the classical limit, and geometry-dynamics correlations.
A. M. Kowalski, M. T. Martin, A. Plastino, O. A. Rosso

Intensive statistical complexity measure of pseudorandom number generators.
H. A. Larrondo, C. M. González, M. T. Martin, A. Plastino, O. A. Rosso

Quantitative EEG analysis of the maturational changes associated with childhood absence epilepsy.

Statistical complexity measure of pseudorandom bit generators.
C. M. González, H. A. Larrondo, O. A. Rosso
Evidence of self-organization in brain electrical activity using wavelet based informational tools.
O. A. Rosso, M. T. Martin, A. Plastino

Characterization of laser propagation through turbulent media by quantifiers based on the wavelet transform.
L. Zunnino, D. Pérez, M. Garavaglia, O. A. Rosso

Order / Disorder in brain electrical activity.
O. A. Rosso, A. Figliola

Intensive entropic non-triviality measure.
P. W. Lamberti, M.T. Martin, A. Plastino, O. A. Rosso

Wavelet analysis can sensitively describe dynamics of ethanol evoked spiky local field potential wave of slug (Limax marginatus) brain.
A. Schütt, I. Ito, O. A. Rosso, A. Figliola

Wavelet statistical complexity analysis of classical limit.
A.M. Kowalski, M.T. Martin, A. Plastino, A. N. Proto, O. A. Rosso

Statistical complexity and disequilibrium.
M.T. Martin, A. Plastino, O. A. Rosso
Elsevier Science, ISSN 0375-9601

Brain electrical activity analysis using wavelet based informational tools (II): Tsallis non-extensivity and complexity measurements.
O. A. Rosso, M. T. Martin, A. Plastino

Wavelet analysis of generalized Tonic-Clonic epileptic seizures.
O. A. Rosso, S. Blanco, A. Rabinowicz

A transient dominance of theta event-related brain potential component characterizes stimulus processing in auditory oddball task.
J. Yordanova, O. A. Rosso, V. Kolev
Clinical Neurophysiology 114 (2003) 529 – 540

A discovery of new features of gastropod local field potentials by application of wavelet tools.
A. Schüett, O. A. Rosso, A. Figliola
Wavelet entropy analysis of event-related potentials indicates modality-independent theta dominance.
J. Yordanova, V. Kolev, O. A. Rosso, M. Schürmann, O. W. Sakowitz, M. Özgören, E. Basar

Brain electrical activity analysis using wavelet based informational tools.
O. A. Rosso, M. T. Martin, A. Plastino

Characterization of time dynamical evolution of electroencephalographic records.
O. A. Rosso, M. L. Mairal
Physica A 312 (2002) 469 – 504

A transient dominance of theta ERP component characterizes passive auditory processing: evidence from developmental study.
V. Kolev, O. A. Rosso, J. Yordanova
Neuroreport 12 (2001) 2791 – 2796

O. A. Rosso, S. Blanco, J. Yordanova, V. Kolev, A. Figliola, M. Schürmann, E. Basar
Journal of Neuroscience Methods 105 (2001) 65 – 75

Wavelet-entropy in event-related potentials: A new method shows ordering of EEG-oscillations.
R. Quian Quiroga, O. A. Rosso, E. Basar, M. Schürmann
Biological Cybernetic 84 (2001) 291 – 299

Time-Frequency analysis of electroencephalogram series (III): information transfer function and wavelets packets.
S. Blanco, A. Figliola, R. Quian Quiroga, O. A. Rosso, E. Serrano

Discrimination measure of correlations in a population of neurons by using the Jensen-Shannon Divergence.
F. Montani, O. A. Rosso, S. R. Schultz
Nonequilibrium Statistical Mechanics and Nonlinear Physics.
XV Conference on Nonequilibrium Statistical Mechanics and Nonlinear Physics.
O. Descalzi, O. A. Rosso, H. A. Larrondo (Editors).

Wavelet analysis of spatiotemporal network oscillations evoked in the Incilaria brain.
Nonequilibrium Statistical Mechanics and Nonlinear Physics.
XV Conference on Nonequilibrium Statistical Mechanics and Nonlinear Physics.
O. Descalzi, O. A. Rosso, H. A. Larrondo (Editors).
Brain maturation changes characterized by Algorithmic Complexity (Lempel and Ziv Complexity).
Nonequilibrium Statistical Mechanics and Nonlinear Physics.
XV Conference on Nonequilibrium Statistical Mechanics and Nonlinear Physics.
O. Descalzi, O. A. Rosso, H. A. Larrondo (Editors).

Complexity as a tool for studying neural activity.
O. A. Rosso, M. T. Martin, A. Plastino
Instabilities and Non-equilibrium Structures X. 2005, in press.
Kluwer Academic Publishers, Dordrecht, Netherlands

A measure of self-organization in neural activity.
O. A. Rosso, M. T. Martin, A. Plastino
Instabilities and Non-equilibrium Structures IX. 2004, pp. 281 – 290

Generalized information measures and the analysis of brain electrical signals.
A. Plastino, M. T. Martin, O. A. Rosso
Nonextensive Entropy - Interdisciplinary applications. 2004, pp. 261 – 293
Edited by: M. Gell-Mann and C. Tsallis.
Santa Fe Institute Studies in the Sciences of Complexity,
Oxford University Press, New York, USA.

Self-organization in neural matter.
M. T. Martin, A. Plastino, O. A. Rosso
Condensed Matter Theories, Vol. 17,
Edited by M. P. Das and F. Green, 2003, pp. 279 - 291
Nova Science Publishers, New York, US.

Time-frequency analysis of sensorial brain activity.
O. A. Rosso, J. Yordanova, V. Kolev, S. Blanco, A. Figliola, E. Basar, M. Schürmann
Advances in Clinical Neurophysiology,
XV International Congress of Clinical Neurophysiology, Buenos Aires, Argentina.
Supplement to Clinical Neurophysiology Vol.54, 2002, pp. 443 – 450
Elsevier Sciences, Amsterdam.

Dynamic characterization of Tonic-Clonic epileptic seizures using wavelet entropy.
O. A. Rosso, S. Blanco, A. Figliola, J. Creso, A. Rabinowicz
Proceedings of the European Medical & Biological Engineering Conference,
EMBEC’99
Wavelet-Entropy: a measure of order in evoked potentials.
R. Quian Quiroga, O. A. Rosso, E. Basar
Edited by C. Barber, G. G. Celesia, I. Hashimoto and R. Kakigi

Wavelet-Entropy applied to brain signal analysis.
O. A. Rosso, R. Quian Quiroga, S. Blanco, A. Figliola, E. Basar
Signal Processing IX, Theory and Applications
Edited by: S. Thodoridis, I. Pitas, A. Stouraitis, N. Kalouptsidis.
Typorama Editions, Athens, Greece.